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(63) c)
$$\begin{cases} x - 2y - 3z - 2t = -19 \\ y + 2z + t = 12 \\ 2y + 3z + t = 16 \\ 3x - 2y + t = 5 \end{cases}$$

$$\begin{pmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 3 & 1 \\ 3 & -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} -19 \\ 12 \\ 16 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -6 & 3 & 1 \\ -3 & -12 & 5 & 1 \\ 3 & 10 & -3 & -1 \\ -3 & -6 & 1 & 1 \end{pmatrix} \begin{pmatrix} -19 \\ 12 \\ 16 \\ 5 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 \\ -2 \\ 10 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 5 \\ 3 \end{pmatrix}$$

X $\quad$ A^{-1} $\quad$ C

$$AX = C \rightarrow \underbrace{A^{-1} \cdot A}_{I} \cdot X = A^{-1} \cdot C$$

$$X = A^{-1} \cdot C$$

(76) (2°) Halla:

a) $\begin{vmatrix} 0 & 4 & -1 \\ 1 & 2 & 1 \\ 3 & 0 & 1 \end{vmatrix} = 12 + 6 - 4 = 14$

b) $\begin{vmatrix} 10 & 47 & 59 \\ 0 & 10 & 21 \\ 0 & 0 & 10 \end{vmatrix} = 1000$

Menor complementario de una matriz

Es el determinante de una submatriz cuadrada que se obtiene suprimiendo filas y columnas de la matriz original.

El rango de una matriz es igual al mayor de los órdenes de los menores no nulos.

(81) (1º) Calcula el rango

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 & -1 & 4 \\ 3 & -1 & 0 & 1 & 1 & 2 \\ 4 & 1 & 3 & 1 & 0 & 6 \\ 7 & 0 & 3 & 2 & 1 & 8 \end{pmatrix}$$

$\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -7 \neq 0$

$3^a = 1^a + 2^a$

$4^a = 2^a + 3^a$

$\text{rang}(A) = 2$

$$B = \begin{pmatrix} 4 & 2 & 1 & 5 & 3 \\ 2 & 3 & 2 & 6 & 5 \\ 6 & 5 & 3 & 12 & 8 \\ 12 & 10 & 6 & 23 & 16 \end{pmatrix}$$

$\begin{vmatrix} 4 & 2 \\ 2 & 3 \end{vmatrix} = 12 - 4 = 8 \neq 0$

$1^a + 2^a + 3^a$

$\begin{vmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 6 & 5 & 3 \end{vmatrix} = 0$

$\begin{vmatrix} 4 & 2 & 5 \\ 2 & 3 & 6 \\ 6 & 5 & 12 \end{vmatrix} = 8 \neq 0$

$\text{rang}(B) = 3$

$$C = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -1 \\ 1 & -1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -1 \neq 0$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2 \neq 0$$

$$\begin{vmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{vmatrix} = 1 \cdot A_{34} = 1(-1)^{7} \begin{vmatrix} 1 & 0 & 0 \\ 1 & -1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2 \neq 0.$$

$$\operatorname{rank}(C) = 4$$

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(43) (25°)

$$\begin{cases} 100z + 10y + x - (100x + 10y + z) = 198 \\ x + y + z = 9 \\ y = \dfrac{x+z}{2} \end{cases}$$

$$\rightarrow -99x + 99z = 198$$

$$\rightarrow 2y = x + z$$

$$\begin{cases} x + y + z = 9 \\ -x + z = 2 \\ x - 2y + z = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ -1 & 0 & 1 & 2 \\ 1 & -2 & 1 & 0 \end{array}\right) \begin{matrix} \\ 1^a + 2^a \\ 2^a \\ 3^a + 2^a \end{matrix} \left(\begin{array}{ccc|c} 0 & 1 & 2 & 11 \\ -1 & 0 & 1 & 2 \\ 0 & -2 & 2 & 2 \end{array}\right)$$

$$\begin{matrix} 1^a = 3^a \\ 2^a \\ 3^a \end{matrix} \left(\begin{array}{ccc|c} 0 & 3 & 0 & 9 \\ -1 & 0 & 1 & 2 \\ 0 & -2 & 2 & 2 \end{array}\right)$$

$$\begin{cases} 3y = 9 \rightarrow y = 3 \ (1^\circ) \\ -x + z = 2 \rightarrow x = 4 - 2 = 2 \\ -2y + 2z = 2 \rightarrow z = \dfrac{2+6}{2} = 4 \ (2^\circ) \end{cases}$$

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(A3) (19°s) Discute y resuelve cuando sea posible

a) $\begin{cases} 2x-3y+z=0 \\ x-ky-3z=0 \\ 5x+2y-z=0 \end{cases}$

$\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 1 & -k & -3 & 0 \\ 5 & 2 & -1 & 0 \end{array}\right) \begin{matrix} 1^a+3^a \\ 2^a+3\cdot 1^a \\ 3^a \end{matrix}$

$\left(\begin{array}{ccc|c} 7 & -1 & 0 & 0 \\ 7 & -(k+9) & 0 & 0 \\ 5 & 2 & -1 & 0 \end{array}\right) \begin{matrix} 1^a-2^a \\ 2^a \\ 3^a \end{matrix} \left(\begin{array}{ccc|c} 0 & k+8 & 0 & 0 \\ 7 & -(k+9) & 0 & 0 \\ 5 & 2 & -1 & 0 \end{array}\right)$

Discusión: $k+8=0 \rightarrow \boxed{k=-8}$ Comp. Indeter.

$\boxed{k\neq -8}$ Comp. Determinado

Resolución:

(*) $\boxed{k=-8}$

$\begin{cases} 7x+y=0 \\ 5x+2y-z=0 \end{cases}$

$y=-7x$

$z=+5x-14x=-9x$

Sol: $\begin{cases} x=\lambda \\ y=-7\lambda \\ z=-9\lambda \end{cases}$

(*) $\boxed{k\neq -8}$

$\begin{cases} (k+8)y=0 \\ 7x-(k+9)y=0 \\ 5x+2y-z=0 \end{cases}$ $\rightarrow y=0$ $\rightarrow x=0$ $\rightarrow z=0$

b) $\begin{cases} 3x+2y-z=1 \\ x-z=1 \\ 2x+2y+kz=0 \end{cases}$

$\left(\begin{array}{ccc|c} 3 & 2 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 2 & 2 & k & 0 \end{array}\right)$ $\begin{matrix} 1^a \\ 3^a - 2\cdot 1^a \\ 3^a - 2\cdot 2^a \end{matrix}$

$\left(\begin{array}{ccc|c} 3 & 2 & -1 & 1 \\ 0 & -2 & -2 & 2 \\ 0 & 2 & k+2 & -2 \end{array}\right)$

$k=0 \rightarrow$ Comp. Indeter

$k\neq 0 \rightarrow$ Comp. Determ.

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$\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 1 & -k & -3 & 0 \\ 5 & 2 & -1 & 0 \end{array}\right)$ $\begin{matrix} 1^a \\ -2\cdot 2^a + 1^a \\ 5\cdot 1^a - 2\cdot 3^a \end{matrix}$ $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 0 & (2k-3) & 7 & 0 \\ 0 & -19 & 7 & 0 \end{array}\right)$

$-19-(2k-3)$

$-16-2k$

$-2(8+k)$

$\begin{matrix} 1^a \\ 2^a \\ 3^a - 2^a \end{matrix}$ $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 0 \\ 0 & (2k-3) & 7 & 0 \\ 0 & -2(8+k) & 0 & 0 \end{array}\right)$

(69) (23°S) $A = \begin{pmatrix} 0 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

$A^2 = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; $A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$I + A + A^2 = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$; $I - A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

$(I + A + A^2)(I - A) = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(69) (19°) Rango de:

$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -6 & 8 \end{pmatrix} \begin{matrix} 1^a \\ 2^a + 2 \cdot 1^a \end{matrix} \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 0 & 16 \end{pmatrix}$

$\text{rang}(A) = 2$

$B = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ $\text{rang}(B) = 2$

$C = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 12 & -24 & 36 \end{pmatrix} \begin{matrix} 1^a \\ 1^a(-2) \\ 2^a(-6) \end{matrix}$ $\text{rang}(C) = 1$

(A1) 26°s

$$\begin{cases} 0'04x + 0'05y + 0'06z = 1050 \\ 0'05x + 0'06y + 0'04z = 950 \\ x + y + z = 20000 \end{cases}$$

$x = 5000$; $y = 5000$; $z = 10000$

CONTROL VIERNES, 23, 8:30

Aula 02.